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1. $f(x) = \ln x, g(x) = ax + \frac{a-1}{x} - 3 (a \in \mathbb{R})$.

$$\varphi(x) = f(x) + g(x)$$
$$a=1 \quad h(x) = f(x) \cdot g(x) \quad \lambda \quad x \quad 2\lambda \geq h(x) \quad \lambda$$
$$\ln 2 \approx 0.6931, \ln 3 \approx 1.0986$$

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$$2\lambda = 0$$

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$$1 \leq a \leq \varphi(x) > 0$$
$$2\|u\|_{L^2(\Omega)}^2 - \frac{2}{3} < H(x)_{\min} < -\frac{1}{2}\|u\|_{L^2(\Omega)}^2, \quad \text{a.e. } x \in \Omega.$$

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$$\square \square \varphi(x) = f(x) + g(x) = \ln x + ax + \frac{a-1}{x} - 3 (x > 0) \square$$
$$\varphi'(x) = \frac{1}{x} + a - \frac{a-1}{x^2} = \frac{ax^2 + x - (a-1)}{x^2} = \frac{[ax - (a-1)](x+1)}{x^2} \quad (x > 0)$$

① $a=0$ $\varphi'(x) > 0$ $x > 0$

② $a > 1$ $\varphi'(x) > 0$ $x > \frac{a-1}{a}$

③ $0 < a < 1$ $\varphi'(x) > 0$ $x > 0$

④ $a=1$ $\varphi'(x) > 0$ $x > 0$

⑤ $a < 0$ $\varphi'(x) > 0$ $0 < x < \frac{a-1}{a}$

$$a < 0 \quad \varphi(x) \quad \left(0, \frac{a-1}{a}\right)$$

$$2 \quad f(x) = x \ln x + kx - 3k$$

$$1 \quad k=1 \quad f(x) \quad (1 \quad f(1))$$

$$2 \quad x > 3 \quad f(x) > 1 \quad k$$

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$$1 \quad 2x - y - 4 = 0$$

$$2 \quad -3$$

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$$1 \quad 2$$

$$1 \quad$$

$$k=1 \quad f(x) = x \ln x + x - 3$$

$$\therefore f(x) = \ln x + 2$$

$$\therefore f(1) = 2 - 2$$

$$\therefore f(x) \quad (1, f(1)) \quad y + 2 = 2(x - 1) \quad 2x - y - 4 = 0$$

$$2 \quad$$

$$f(x) > 1 \quad x \ln x + kx - 3k > 1 \quad k(x - 3) > 1 - x \ln x$$

$$x > 3 \quad \therefore k > \frac{1 - x \ln x}{x - 3}$$

$$g(x) = \frac{1 - x \ln x}{x - 3} \quad \therefore g(x) = \frac{3 \ln x - x + 2}{(x - 3)^2}$$

$$h(x) = 3 \ln x - x + 2 \quad h(x) = \frac{3 - x}{x} < 0$$

$$\therefore h(x) \quad (3, +\infty)$$

$$h(8) = 3 \ln 8 - 6 > 0 \quad h(9) = 3 \ln 9 - 7 < 0$$

$$\therefore \exists x_0 \in (8, 9) \quad h(x_0) = 0 \quad 3 \ln x_0 - x_0 + 2 = 0 \quad \ln x_0 = \frac{x_0 - 2}{3}$$

$$m(x_0) = 0 \quad h(x)_{\min} = h(x_0) \quad h(x_0) \quad k.$$

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$$k = -1 \quad f(x) = xe^x \quad f'(x) = (x+1)e^x$$

$$\therefore x \in (-\infty, -1) \quad f'(x) < 0 \quad x \in (-1, +\infty) \quad f'(x) > 0$$

$$\therefore f(x) \quad (-\infty, -1) \quad (-1, +\infty)$$

$$\therefore f(x) \quad f(-1) = -\frac{1}{e}$$

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$$g(x) = (x-k-1)e^x + e^2 \quad g'(x) = (x-k)e^x$$

$$\therefore x \in (-\infty, k) \quad g'(x) < 0 \quad x \in (k, +\infty) \quad g'(x) > 0$$

$$\therefore g(x) \quad (-\infty, k) \quad (k, +\infty)$$

$$\textcircled{1} \quad k \leq 0 \quad g(x) \quad (0, +\infty) \quad g(x) \quad (0, +\infty) \quad g(0) < 0$$

$$-k-1+e^2 < 0 \quad k > e^2-1 > 0$$

$$\textcircled{2} \quad k > 0 \quad g(x) \quad (0, k) \quad (k, +\infty)$$

$$g(k) > 0 \quad 0 < k < 2 \quad g(x)_{\min} = g(k) > 0 \quad g(x) \quad (0, +\infty)$$

$$g(k) = 0 \quad k = 2 \quad g(x) \quad (0, +\infty) \quad x = 2$$

$$g(k) < 0 \quad k > 2 \quad g(k+1) = e^2 > 0 \quad g(k) \quad g(k+1) < 0$$

$$\therefore g(x) \quad (k, k+1) \quad g(0) = -k-1+e^2 \leq 0 \quad k \geq e^2-1$$

$$k = 2 \quad k \geq e^2-1 \quad g(x) \quad (0, +\infty)$$

$$k \quad 2 \cup [e^2-1, +\infty)$$

3. 证明：当 $x \in (1, +\infty)$ 时， $x \ln x + x > k(x-1)$ 恒成立。

证明：

1. 构造函数

2. 求导

3. 分析

证明：

(1) 构造函数

(2) 求导 $f'(x) = \ln x + 1$ ，分析导函数的符号。

(3) 证明 $k \leq \frac{x \ln x + x}{x-1}$ 恒成立。令 $g(x) = \frac{x \ln x + x}{x-1}$ ，分析 $g(x)$ 在 $(1, +\infty)$ 上的最小值。

1.

$$f(x) = x - \ln x - 2$$

$$\therefore f(1) = -1, f'(x) = 1 - \frac{1}{x}$$

$$\therefore f'(1) = 0$$

$$\therefore f(x) \text{ 在 } (1, 1) \text{ 处取得极小值 } y = -1$$

2.

$$f(x) = x - \ln x - 2$$

$$\therefore f'(x) = 1 - \frac{1}{x}$$

$$\text{当 } x \in (3, 4) \text{ 时， } f'(x) = 1 - \frac{1}{x} > 0$$

$$\therefore f(x) \text{ 在 } (3, 4) \text{ 上单调递增}$$

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□1□ $f(x) = \frac{4x^2 - ax + 1}{x}$ □□□□ $4x^2 - ax + 1 = 0$ □ $\Delta = a^2 - 16$ □□□□ $\Delta \leq 0$ □ $\Delta > 0$ □ $f(x)$ □□□□□□□□□□□□

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□2□□□□ $h(x) = \frac{x + x \ln x}{x - 1}$ □□□□ $h(x) = \frac{x - \ln x - 2}{(x - 1)^2} (x > 1)$ □□□□□□ $h(x) = x - \ln x - 2 (x > 1)$ □□□ $h(x)$ □□□□□□

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□1□ $f(x)$ □□□□□□ $(0, +\infty)$ □□ $f(x) = \frac{1}{x} + 4x - a = \frac{4x^2 - ax + 1}{x}$ □

□□□□ $4x^2 - ax + 1 = 0$ □ $\Delta = a^2 - 16$ □

①□ $\Delta = a^2 - 16 \leq 0$ □□ $0 < a \leq 4$ □□ $4x^2 - ax + 1 \geq 0$ □□□□

□□ $f(x) \geq 0$ □□ $x \in (0, +\infty)$ □□□□

□□ $f(x)$ □ $(0, +\infty)$ □□□□□□□□□□□□□□□□ 0□

②□ $\Delta = a^2 - 16 > 0$ □□ $a > 4$ □□

□□□ $4x^2 - ax + 1 = 0$ □□□□□□□□ x_1 □ x_2 □

□ $x_1 + x_2 = \frac{a}{4}$ □ $x_1 x_2 = \frac{1}{4}$ □□□□ $x_1 > 0$ □ $x_2 > 0$ □□ $x_1 < x_2$ □

□ $x_1 = \frac{a - \sqrt{a^2 - 16}}{8}$ □ $x_2 = \frac{a + \sqrt{a^2 - 16}}{8}$ □

□ $f(x) > 0$ □ $4x^2 - ax + 1 > 0$ □□□□ $0 < x < \frac{a - \sqrt{a^2 - 16}}{8}$ □ $x > \frac{a + \sqrt{a^2 - 16}}{8}$ □

6. 设函数 $f(x) = \frac{1}{a}x^2 + \ln x - \left(2 + \frac{1}{a}\right)x$ ($a \neq 0$)

(1) 讨论 $f(x)$ 的单调性

(2) 若 $F(x) = af(x) - x^2$, $F(x) < 1 - 2ax$, $x \in (1, +\infty)$, 求 a 的取值范围

($\ln 3 < \frac{4}{3} < \ln 4 < \frac{5}{4}$)

解：(1) ① 当 $a > 0$ 时，

② 当 $a < 0$ 时，

(1) $f'(x) = \frac{(x-a)(2x-1)}{ax}$ ① 当 $a > 0$ 时，

(2) ① 当 $a < 0$ 时， $f'(x) = \frac{(x-a)(2x-1)}{ax}$ ② 当 $a > 0$ 时， $f'(x) = \frac{(x-a)(2x-1)}{ax}$

$a < h(x)_{\min}$ ③ 当 $a > 0$ 时，

④ 当 $a < 0$ 时，

(1) $f(x)$ 在 $(0, +\infty)$ 上 $f'(x) = \frac{2x}{a} + \frac{1}{x} - \left(2 + \frac{1}{a}\right) = \frac{2x^2 - (2a+1)x + a}{ax} = \frac{(x-a)(2x-1)}{ax}$

① 当 $a < 0$ 时， $f'(x) > 0$ ② 当 $0 < x < \frac{1}{2}$ 时，

$\therefore a < 0$ 时， $f(x)$ 在 $\left(0, \frac{1}{2}\right)$ 上单调递增，在 $\left(\frac{1}{2}, +\infty\right)$ 上单调递减。

② 当 $0 < a < \frac{1}{2}$ 时， $f'(x) > 0$ ③ 当 $0 < x < a$ 时， $x > \frac{1}{2}$ 时，

$\therefore f(x)$ 在 $(0, a)$ 上单调递增，在 $\left(\frac{1}{2}, +\infty\right)$ 上单调递减。

③ 当 $a = \frac{1}{2}$ 时， $f'(x) \geq 0$ ④ 当 $f(x)$ 在 $(0, +\infty)$ 上单调递增。

$$\textcircled{4} \quad a > \frac{1}{2} \quad f'(x) > 0 \quad 0 < x < \frac{1}{2} \quad x > a$$

$$\therefore f(x) \text{ 在 } \left(0, \frac{1}{2}\right) \text{ 上 递增, 在 } \left(\frac{1}{2}, a\right) \text{ 上 递减}$$

$$(2) \quad F(x) = af(x) - x^2 = \ln x - (2a+1)x \quad F(x) < 1 - 2ax \Leftrightarrow a < \frac{x+1}{\ln x}, (x > 1)$$

$$h(x) = \frac{x+1}{\ln x}, (x > 1) \quad h'(x) = \frac{\ln x - \frac{1}{x} - 1}{(\ln x)^2}$$

$$h'(x) = \ln x - \frac{1}{x} - 1 \quad (x > 1) \quad h'(x) \in (1, +\infty) \quad h'(3) < 0 \quad h'(4) > 0$$

$$\therefore \exists x_0 \in (3, 4) \quad h(x_0) = \ln x_0 - \frac{1}{x_0} - 1 = 0 \quad h(x) \text{ 在 } (1, x_0] \text{ 上 递减, 在 } [x_0, +\infty) \text{ 上 递增}$$

$$\therefore h(x)_{\min} = h(x_0) = \frac{x_0+1}{\ln x_0} = \frac{x_0+1}{\frac{1}{x_0}+1} = x_0 \in (3, 4)$$

$$\therefore a < h(x)_{\min} \quad a \text{ 的取值范围是 } (3, 4)$$

解法二

$$a < \frac{x+1}{\ln x}, (x > 1) \quad a < \frac{x+1}{\ln x} \quad a < \frac{x+1}{\ln x}.$$

$$f(x) = \ln x - \frac{1}{2}ax^2 + (a-1)x \quad a \in \mathbf{R}$$

$$f'(x) = \frac{1}{x} - ax + (a-1)$$

$$f(x) \leq \frac{e^x}{2e^2} - \frac{1}{2}ax^2 - x \quad a \text{ 的取值范围}$$

$$1 \leq a \leq 2 \quad 1 \leq a \leq 2$$

解法二

$$1 \leq a \leq 2 \quad a \text{ 的取值范围}$$

$$2 \leq a \leq \frac{e^x - \ln x}{x} \quad g(x) = \frac{e^x - \ln x}{x} \quad g(x) \text{ 的取值范围}$$

解法二

$$\square 1 \square \quad f(x) \quad (0, +\infty)$$

$$f(x) = \frac{1}{x} - ax + a - 1 = \frac{-ax^2 + (a-1)x + 1}{x} = \frac{(-ax-1)(x-1)}{x}$$

$$\square 1 \square \quad a \geq 0 \square \quad -ax-1 < 0 \square$$

$$\square \quad f(x) < 0 \quad x > 1 \square$$

$$\square \quad f(x) > 0 \quad 0 < x < 1 \square$$

$$\therefore f(x) \quad (0, 1) \quad (1, +\infty)$$

$$\square 2 \square \quad -1 < a < 0 \square \quad \frac{1}{-a} > 1 \square$$

$$\square \quad f(x) > 0 \quad 0 < x < 1 \square \quad x > \frac{1}{-a} \square$$

$$\square \quad f(x) < 0 \quad 1 < x < -\frac{1}{a} \square$$

$$\therefore f(x) \quad \left(1 - \frac{1}{a}\right) \quad (0, 1) \quad \left(-\frac{1}{a}, +\infty\right)$$

$$\square 3 \square \quad a = -1 \square \quad \frac{1}{-a} = 1 \square \quad f(x) \geq 0 \quad (0, +\infty) \square$$

$$\therefore f(x) \quad (0, +\infty)$$

$$\square 4 \square \quad a < -1 \square \quad 0 < \frac{1}{-a} < 1 \square$$

$$\square \quad f(x) > 0 \quad 0 < x < \frac{1}{-a} \square \quad x > 1 \square$$

$$\square \quad f(x) < 0 \quad -\frac{1}{a} < x < 1 \square$$

$$\therefore f(x) \quad \left(-\frac{1}{a}, 1\right) \quad \left(0, -\frac{1}{a}\right) \quad (1, +\infty) \square$$

$$\square\square\square\square\square\square a < -1 \square\square f(x) \square\square\square\square\square\square \left(-\frac{1}{a}, 1\right) \square\square\square\square\square\square \left(0, -\frac{1}{a}\right) \square (1, +\infty) \square$$

$$\square a = -1 \square\square f(x) \square\square\square\square\square\square (0, +\infty) \square\square\square\square\square\square$$

$$\square -1 < a < 0 \square\square f(x) \square\square\square\square\square\square \left(1, -\frac{1}{a}\right) \square\square\square\square\square\square (0, 1) \square \left(-\frac{1}{a}, +\infty\right)$$

$$\square a \geq 0 \square\square f(x) \square\square\square\square\square\square (0, 1) \square\square\square\square\square\square (1, +\infty) \square$$

$$\square 2 \square f(x) = \ln x - \frac{1}{2}ax^2 + (a-1)x \leq \frac{e^x}{2e^2} - \frac{1}{2}ax^2 - x \Leftrightarrow \ln x + ax \leq \frac{e^x}{2e^2} \Leftrightarrow a \leq \frac{\frac{e^x}{2e^2} - \ln x}{x}$$

$$\square g(x) = \frac{\frac{e^x}{2e^2} - \ln x}{x} \square\square g'(x) = \frac{\frac{1}{2e^2}(x-1)e^x - 1 + \ln x}{x^2} \square$$

$$\square h(x) = \frac{1}{2e^2}(x-1)e^x - 1 + \ln x \square\square h'(x) = \frac{1}{2e^2}xe^x + \frac{1}{x} > \square\square\square$$

$$\therefore h(x) \square (0, +\infty) \square\square\square\square\square\square$$

$$\therefore h(1) = -1 < 0$$

$$h(2) = \frac{1}{2} - 1 + \ln 2 > \frac{1}{2} - 1 + \ln \sqrt{e} = \frac{1}{2} - 1 + \frac{1}{2} = 0$$

$$\therefore \exists x_0 \in (1, 2) \square\square h(x_0) = \frac{1}{2e^2}(x_0 - 1)e^{x_0} - 1 + \ln x_0 = 0 \square$$

$$- \ln x_0 = \frac{1}{2e^2}(x_0 - 1)e^{x_0} - 1$$

$$x \in (0, x_0) \square h(x) < 0 \square\square\square g'(x) < 0 \square$$

$$\therefore x \in (0, x_0) \square\square g'(x) < 0 \square g(x) \square (0, x_0) \square\square\square\square\square\square$$

$$x \in (x_0, +\infty) \square\square h(x) > 0 \square\square\square g'(x) > 0 \square$$

$$\therefore x \in (x_0, +\infty) \square\square g'(x) \square x \in (x_0, +\infty) \square\square\square\square\square\square$$

$$\therefore g(x)_{\min} = g(x_0) = \frac{e^{x_0} - \ln x_0}{x_0} - \ln x_0 = \frac{1}{2e^2} (x_0 - 1) e^{x_0} - 1$$

$$g(x_0) = \frac{\frac{e^{x_0}}{2e^2} + \frac{1}{2e^2} (x_0 - 1) e^{x_0} - 1}{x_0} = \frac{e^{x_0}}{2e^2} - \frac{1}{x_0}$$

$$p(x) = \frac{e^x}{2e^2} - \frac{1}{x} \quad (x \in (1, 2))$$

$$p(x)$$

$$p(1) < p(x) < p(2) \quad p(1) = \frac{1}{2e} - 1 \in (-1, 0) \quad p(2) = 0$$

$$g(x_0) \in (-1, 0)$$

$$\therefore a$$

$$a$$

$$g(x)_{\min} = \frac{e^{x_0}}{2e^2} - \frac{1}{x_0} \quad p(x) = \frac{e^x}{2e^2} - \frac{1}{x} \quad (x \in (1, 2)) \quad p(x)$$

$$f(x)$$

$$f(x) = a \ln x - (2a+1)x$$

$$f(x)$$

$$f(x) < 1 - 2ax \quad x \in (1, +\infty) \quad a \quad (\ln 3 < \frac{4}{3} < \ln 4 < \frac{5}{4}).$$

$$1$$

$$a$$

$$1$$

$$a < \frac{x+1}{\ln x} \quad x \in (1, +\infty) \quad h(x) = \frac{x+1}{\ln x} \quad (x > 1) \quad h(x) \quad a$$

$$a$$

$$f'(x) = \frac{a}{x} - (2a+1) = \frac{a - (2a+1)x}{x}.$$

$$a < -\frac{1}{2} \implies f(x) > 0 \implies x > \frac{a}{2a+1} \implies f(x) < 0 \implies 0 < x < \frac{a}{2a+1}$$

$$f(x) \text{ is increasing on } \left(0, \frac{a}{2a+1}\right) \text{ and decreasing on } \left(\frac{a}{2a+1}, +\infty\right).$$

$$-\frac{1}{2} \leq a \leq 0 \implies f(x) < 0 \text{ for all } x \in (0, +\infty)$$

$$a > 0 \implies f(x) > 0 \implies 0 < x < \frac{a}{2a+1} \implies f(x) < 0 \implies x > \frac{a}{2a+1}$$

$$f(x) \text{ is increasing on } \left(0, \frac{a}{2a+1}\right) \text{ and decreasing on } \left(\frac{a}{2a+1}, +\infty\right).$$

$$f(x) < 1 - 2ax \implies x \in (1, +\infty) \implies a < \frac{x+1}{\ln x} \implies x \in (1, +\infty)$$

$$h(x) = \frac{x+1}{\ln x} \quad (x > 1) \implies h'(x) = \frac{\ln x - \frac{1}{x} - 1}{(\ln x)^2}.$$

$$h'(x) = \ln x - \frac{1}{x} - 1 \quad (x > 1)$$

$$\because y = \ln x \implies y' = \frac{1}{x} \implies (1, +\infty)$$

$$\therefore h'(x) \in (1, +\infty) \implies h'(3) = \ln 3 - \frac{4}{3} < 0 \implies h'(4) = \ln 4 - \frac{5}{4} > 0.$$

$$\therefore \exists x_0 \in (3, 4) \implies h(x_0) = \ln x_0 - \frac{1}{x_0} - 1 = 0 \implies \ln x_0 = \frac{1}{x_0} + 1$$

$$\therefore \implies x \in (1, x_0) \implies h'(x) < 0 \implies x \in (x_0, +\infty) \implies h'(x) > 0$$

$$\therefore h(x) \text{ is decreasing on } (1, x_0) \text{ and increasing on } (x_0, +\infty)$$

$$\therefore h(x)_{\min} = h(x_0) = \frac{x_0+1}{\ln x_0} = \frac{x_0+1}{\frac{1}{x_0}+1} = x_0 \in (3, 4)$$

$$\therefore f(x) \text{ 在 } (a, +\infty) \text{ 上单调递增.}$$

$$\text{3. 当 } x \in (2, +\infty) \text{ 时, } f(x) > 0$$

$$(x-1)e^x + a(2e - e^x) > 0$$

$$\text{当 } x \in (2, +\infty) \text{ 时, } \frac{(x-1)e^x}{e^x - 2e} > a$$

$$\left[\frac{(x-1)e^x}{e^x - 2e} \right]_{\min} > a, \quad x \in (2, +\infty)$$

$$g(x) = \frac{(x-1)e^x}{e^x - 2e}, \quad x \in (2, +\infty)$$

$$g'(x) = \frac{e^x(e^x - 2ex)}{(e^x - 2e)^2}$$

$$h(x) = e^x - 2ex, \quad x \in (2, +\infty)$$

$$h(x) = e^x - 2e > 0$$

$$\therefore h(x) = e^x - 2ex \text{ 在 } (2, +\infty) \text{ 上单调递增.}$$

$$h(2) = e^2 - 4e < 0, \quad h(3) = e^3 - 6e > 0$$

$$\therefore g'(x) \text{ 在 } (2, 3) \text{ 上存在唯一零点 } x_0, \text{ 使得 } e^{x_0} = 2ex_0, \quad x_0 \in (2, 3)$$

$$\therefore g(x) \text{ 在 } (2, x_0) \text{ 上单调递减, 在 } (x_0, +\infty) \text{ 上单调递增}$$

$$\therefore g(x)_{\min} = g(x_0) = \frac{(x_0 - 1)e^{x_0}}{e^{x_0} - 2e} = \frac{(x_0 - 1)2ex_0}{2ex_0 - 2e}, \quad x_0 \in (2, 3)$$

$$\therefore a < x_0 \in (2, 3)$$

$$\text{即 } a \text{ 在 } (2, 3) \text{ 内.}$$

$$f(1) = a - 1 = -1 \quad a = 0$$

$$f(x) = \frac{ax - 1 + \ln x}{x^2}$$

$$g(x) = ax - 1 + \ln x \quad g'(x) = \frac{1}{x} + a$$

$$a \geq 0 \quad g'(x) > 0 \quad y = g(x)$$

$$a < 0 \quad x \in (0, \frac{1}{a}) \quad g'(x) > 0 \quad y = g(x)$$

$$x \in (-\frac{1}{a}, +\infty) \quad g'(x) < 0 \quad y = g(x)$$

$$x = -\frac{1}{a} \quad g(x)_{\min} = g(-\frac{1}{a}) = \ln(-\frac{1}{a}) - 2$$

$$g(x) \quad \ln(-\frac{1}{a}) - 2 > 0 \quad -e^{-2} < a < 0$$

$$-e^{-2} < a < 0 \quad -\frac{1}{a} > e^2 > 1$$

$$g(1) = a - 1 < 0 \quad y = g(x) \quad (0, \frac{1}{a})$$

$$-e^{-2} < a < 0 \quad (-\frac{1}{a})^2 > -\frac{1}{a}$$

$$g(-\frac{1}{a}) = \ln(-\frac{1}{a})^2 + \frac{1}{a} - 1 \quad t = -\frac{1}{a} \quad y = 2\ln t - t - 1 (t > e)$$

$$y' = \frac{2-t}{t} < 0 \quad y = 2\ln t - t - 1 (t > e)$$

$$y < 2\ln(e^2) - e^2 - 1 = 3 - e^2 < 0 \quad g(-\frac{1}{a}) = \ln(-\frac{1}{a})^2 + \frac{1}{a} - 1 < 0$$

$$y = g(x) \quad (-\frac{1}{a}, +\infty)$$

$$a \quad (-e^{-2}, 0)$$

$$a = 2 \quad f(x) = (2 - \frac{1}{x})\ln x \quad f'(x) = \frac{1}{x}\ln x + (2 - \frac{1}{x})\frac{1}{x} = \frac{2x - 1 + \ln x}{x^2}$$

$$h(x) = 2x - 1 + \ln x \quad h'(x) = \frac{1}{x} + 2 > 0 \quad y = h(x)$$

$$\square H(x) = f(x) - \frac{1}{x} \square\square\square H(x) = f(x) - \frac{1}{x} \square(0, 2] \square\square\square\square.$$

$$\square\square H(x) = \ln x - k + \frac{1}{x^2} \geq 0 \square(0, 2] \square\square\square\square$$

$$\square k \leq \ln x + \frac{1}{x^2} \square(0, 2] \square\square\square\square\square\square u(x) = \ln x + \frac{1}{x^2} \square\square\square\square\square k \square\square\square\square.$$

$$\square 3 \square\square\square\square\square\square x \in \left[\frac{1}{e}, e^2 \right] \square\square\square f(x) > 3 \ln x \square\square\square$$

$$\square\square (\ln x - k - 1) x > 3 \ln x \square\square\square\square x \in \left[\frac{1}{e}, e^2 \right] \square\square\square\square$$

$$\square k + 1 < \frac{(x-3)\ln x}{x} \square\square\square\square x \in \left[\frac{1}{e}, e^2 \right] \square\square\square\square$$

$$\square g(x) = \frac{(x-3)\ln x}{x} \square\square\square\square\square\square\square\square g(x) \square\square\square\square\square\square\square\square k \square\square\square\square.$$

$$\square\square\square$$

$$\square 1 \square\square\square\square\square\square f(x) = \ln x - k \square$$

$$\square\square\square y = f(x) \square(1, f(1)) \square\square\square\square\square\square\square y = 3x \square\square\square$$

$$\square\square f(1) = \ln 1 - k = 3 \square\square\square k = -3.$$

$$\square 2 \square\square\square f(x_1) - f(x_2) < \frac{1}{x_1} - \frac{1}{x_2} \square\square\square f(x_1) - \frac{1}{x_1} < f(x_2) - \frac{1}{x_2} \square$$

$$\square H(x) = f(x) - \frac{1}{x} \square\square\square\square x_1, x_2 \in (0, 2] \square x_1 < x_2 \square$$

$$\square\square H(x) = f(x) - \frac{1}{x} \square(0, 2] \square\square\square\square\square.$$

$$\square\square H(x) = \ln x - k + \frac{1}{x^2} \geq 0 \square(0, 2] \square\square\square\square\square$$

$$\square k \leq \ln x + \frac{1}{x^2} \square(0, 2] \square\square\square\square\square\square u(x) = \ln x + \frac{1}{x^2} \square$$

$$\square \square \quad u'(x) = \frac{1}{x} - \frac{2}{x^3} = \frac{x^2 - 2}{x^3} \quad \square \square \quad u'(x) = \frac{x^2 - 2}{x^3} = 0 \quad \square \square \square \quad x = \sqrt{2} \quad \square$$

$$\square \square \square \quad 0 < x < \sqrt{2} \quad \square \square \quad f'(x) < 0 \quad \square \quad f(x) \quad \square \square \square \square$$

$$\square \quad \sqrt{2} < x < 2 \quad \square \square \quad f'(x) > 0 \quad \square \quad f(x) \quad \square \square \square \square$$

$$\square \square \square \quad x = \sqrt{2} \quad \square \square \quad f(x) \quad \square \square \square \square \quad \ln \sqrt{2} + \frac{1}{2} \quad \square$$

$$\square \square \quad k \leq \ln \sqrt{2} + \frac{1}{2}.$$

$$\square \square \square \square \square \square \square \quad x \in \left[\frac{1}{e}, e^2 \right] \quad \square \square \square \quad f(x) > 3 \ln x \quad \square \square \square$$

$$\square \square \quad (\ln x - k - 1) x > 3 \ln x \quad \square \square \square \square \quad x \in \left[\frac{1}{e}, e^2 \right] \quad \square \square \square \square$$

$$\square \quad k + 1 < \frac{(x - 3) \ln x}{x} \quad \square \square \square \square \quad x \in \left[\frac{1}{e}, e^2 \right] \quad \square \square \square \square$$

$$\square \quad g'(x) = \frac{(x - 3) \ln x}{x} \quad \square \square \square \quad g'(x) = \frac{3 \ln x + x - 3}{x^2} \quad \square$$

$$\square \square \quad u'(x) = 3 \ln x + x - 3 \quad \square \square \square \quad u'(x) = \frac{3}{x} + 1 > 0 \quad \square \quad x \in \left[\frac{1}{e}, e^2 \right] \quad \square \square \square \square$$

$$\square \square \quad u'(x) = 3 \ln x + x - 3 \quad \square \quad x \in \left[\frac{1}{e}, e^2 \right] \quad \square \square \square \square \square \square$$

$$\square \quad u(2) = 3 \ln 2 + 2 - 3 = 3 \ln 2 - 1 > 0 \quad \square \quad u\left(\frac{3}{2}\right) = 3 \ln \frac{3}{2} - \frac{3}{2} = 3 \left(\ln \frac{3}{2} - 1 \right) < 0 \quad \square$$

$$\square \square \square \square \square \square \square \square \quad x_0 \in \left(\frac{3}{2}, 2 \right) \quad \square \square \square \quad u(x_0) = 3 \ln x_0 + x_0 - 3 = 0 \quad \square$$

$$\square \quad \ln x_0 = \frac{3 - x_0}{3} \quad \square$$

$$\text{当 } \frac{1}{e} < x < x_0 \text{ 时 } f'(x) < 0 \text{ 且 } f'(x) \text{ 单调递增}$$

$$\text{当 } x_0 < x < e^2 \text{ 时 } f'(x) > 0 \text{ 且 } f'(x) \text{ 单调递减}$$

$$\text{当 } g(x)_{\min} = g(x_0) = \frac{(x_0 - 3) \ln x_0}{x_0} = \frac{(x_0 - 3) \frac{3 - x_0}{3}}{x_0} = 2 - \frac{1}{3} \left(x_0 + \frac{9}{x_0} \right)$$

$$\text{当 } x_0 \in \left(\frac{3}{2}, 2 \right) \text{ 时 } -\frac{1}{2} < 2 - \frac{1}{3} \left(x_0 + \frac{9}{x_0} \right) < -\frac{1}{6}$$

$$\text{当 } k \in \mathbb{Z} \text{ 时 } k+1 \text{ 与 } k-1$$

$$\text{当 } k \text{ 为偶数时 } -2.$$

$$\text{当 } x \in (0, 4) \text{ 时 } f(x) = \ln x + 1 + \frac{2a}{x} \text{ 在 } (a, f(a)) \text{ 处取得极小值}$$

$$13 \text{ 当 } x \in (0, 4) \text{ 时 } f(x) = \ln x + 1 + \frac{2a}{x} \text{ 在 } (a, f(a)) \text{ 处取得极小值}$$

$$(1) \text{ 当 } a \text{ 为任意实数时 } f(x) \text{ 在 } (a, f(a)) \text{ 处取得极小值}$$

$$(2) \text{ 当 } k \text{ 满足 } 2f(x) > k \left(1 - \frac{1}{x} \right) \text{ 对 } x \in (1, +\infty) \text{ 恒成立时 } k \text{ 的取值范围}$$

$$\text{当 } a=1 \text{ 时 } f(x) \text{ 在 } (0, 2) \text{ 处取得极大值, 在 } (2, +\infty) \text{ 处取得极小值}$$

$$\text{当 } a=1 \text{ 时 } f(x) \text{ 在 } (0, 2) \text{ 处取得极大值, 在 } (2, +\infty) \text{ 处取得极小值}$$

$$\text{当 } a=1 \text{ 时 } f(x) \text{ 在 } (0, 2) \text{ 处取得极大值, 在 } (2, +\infty) \text{ 处取得极小值}$$

$$\text{当 } a=1 \text{ 时 } f(x) \text{ 在 } (0, 2) \text{ 处取得极大值, 在 } (2, +\infty) \text{ 处取得极小值}$$

$$\text{当 } 2f(x) > k \left(1 - \frac{1}{x} \right) \text{ 对 } x \in (1, +\infty) \text{ 恒成立时 } k < \frac{2 \left(\ln x + 1 + \frac{2}{x} \right)}{1 - \frac{1}{x}} \text{ 恒成立}$$

$$g'(x) = \frac{2(x - \ln x - 4)}{(x-1)^2} \text{ 当 } \varphi(x) = x - \ln x - 4 \text{ 在 } (1, +\infty) \text{ 上单调递增时 } x_0 \in (5.5, 6) \text{ 且 } \varphi(x_0) = 0$$

$$\text{当 } g(x)_{\min} = g(x_0) \text{ 时 } g(x) \text{ 在 } (x_0, +\infty) \text{ 上单调递增}$$

$$\text{当 } a=1 \text{ 时 } f(x) \text{ 在 } (0, 2) \text{ 处取得极大值, 在 } (2, +\infty) \text{ 处取得极小值}$$

$$(1) f(x) \text{ 在 } (0, +\infty) \text{ 上 } f'(x) = \frac{1}{x} - \frac{2a}{x^2} = \frac{x-2a}{x^2} \therefore x=a \text{ 时 } f'(a) = \frac{a-2a}{a^2} = -\frac{1}{a}$$

$$\therefore y=f(a) = -\frac{1}{a}(x-a) \text{ 即 } y=\ln a - 1 - \frac{2a}{a} = -\frac{1}{a}(x-a)$$

$$\therefore \text{在 } (0, 4) \text{ 上 } a=1 \therefore f'(x) = \frac{x-2}{x^2}$$

$$\text{在 } f(x) \text{ 在 } (0, 2) \text{ 上 } \text{在 } (2, +\infty) \text{ 上}$$

$$\therefore \text{在 } x \in (1, +\infty) \text{ 上 } 1 - \frac{1}{x} > 0 \therefore 2f(x) > k \left(1 - \frac{1}{x} \right) \text{ 即 } k < \frac{2 \left(\ln x + 1 + \frac{2}{x} \right)}{1 - \frac{1}{x}}$$

$$\text{即 } g(x) = \frac{2 \left(\ln x + 1 + \frac{2}{x} \right)}{1 - \frac{1}{x}} = \frac{2(x \ln x + x + 2)}{x-1} \therefore g'(x) = \frac{2(x - \ln x - 4)}{(x-1)^2}$$

$$\text{在 } \varphi(x) = x - \ln x - 4 \text{ 上 } \varphi'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} > 0 \therefore \varphi(x) \text{ 在 } (1, +\infty) \text{ 上}$$

$$\therefore \varphi(5.5) = 1.5 - \ln 5.5 = \ln e^{\frac{3}{2}} - \ln \frac{11}{2} \text{ 即 } e^{\frac{3}{2}} < 3^3 = 27 \text{ 即 } \left(\frac{11}{2} \right)^2 = \frac{121}{4} = 30.25 \text{ 即 } e^3 < \left(\frac{11}{2} \right)^2$$

$$\text{即 } e^{\frac{3}{2}} < \frac{11}{2} \text{ 即 } \varphi(5.5) < 0$$

$$\text{在 } \varphi(6) = 2 - \ln 6 = \ln e^2 - \ln 6 > \ln 2.5^2 - \ln 6 = \ln 6.25 - \ln 6 > 0$$

$$\text{在 } x_0 \in (5.5, 6) \text{ 上 } \varphi(x_0) = 0 \text{ 即 } x_0 - \ln x_0 - 4 = 0 \text{ ①}$$

$$\text{在 } x \in (1, x_0) \text{ 上 } g'(x) < 0 \text{ 在 } x \in (x_0, +\infty) \text{ 上 } g'(x) > 0$$

$$\text{在 } x \in (1, x_0) \text{ 上 } g(x) \text{ 在 } x \in (x_0, +\infty) \text{ 上 } g(x) \text{ 在 } x_0 \text{ 处取得最小值}$$

$$\therefore g(x)_{\min} = g(x_0) = \frac{2(x_0 \ln x_0 + x_0 + 2)}{x_0 - 1}$$

① $g(x)_{\min} = g(x_0) = \frac{2[x_0(x_0 - 4) + x_0 + 2]}{x_0 - 1} = 2(x_0 - 2) \in (7, 8)$

k 7

1

2 $f(x) > 0$ $f(x)_{\min} > 0$ $f(x) < 0$ $\Leftrightarrow f(x)_{\max} < 0$

3 $f(x) > g(x)$ $f(x)_{\min} > g(x)_{\max}$

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